Express Analysis of Economical Time Series

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Volzhskij Utes, November 26

Outline

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Modern Approach to Data Processing

Clear Mathematical Models

- Each step is clear and elementary
- Local changes of data or updates can be processed locally

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Fuzzy Sets – Sets with Flexible Boundaries

Crisp Set of all real numbers which lie within $[0.874, 1.093]$

Fuzzy Set of real numbers which are "approximately equal to" 1

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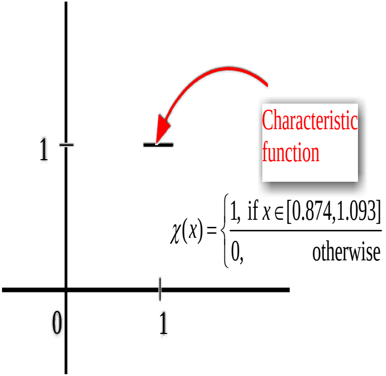

$$\chi(x) = \begin{cases} 1, & \text{if } x \in [0.874, 1.093] \\ 0, & \text{otherwise} \end{cases}$$

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Fuzzy Sets – Sets with Flexible Boundaries

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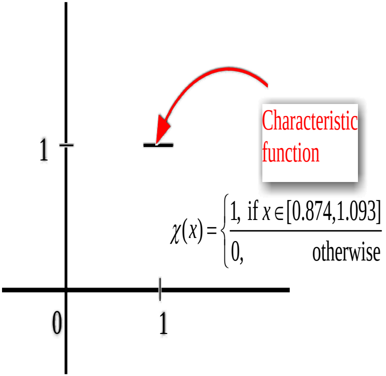

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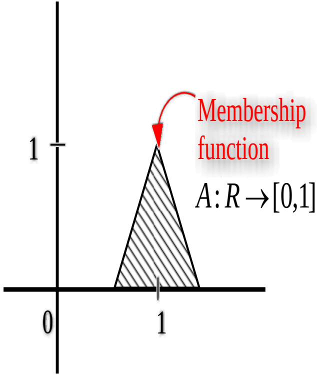

Fuzzy Partition $A_1, \dots, A_n$ of $[a, b]$

Fuzzy sets $A_1, \dots, A_n$ with continuous membership functions form a **fuzzy partition** with nodes $x_1, \dots, x_n$ if for each $k = 1 \dots, n$

- $A_k(x_k) = 1$
- $A_k(x) = 0$ if $x \notin (x_{k-1}, x_{k+1})$
- $A_k(x) \nearrow$ on $[x_{k-1}, x_k]$
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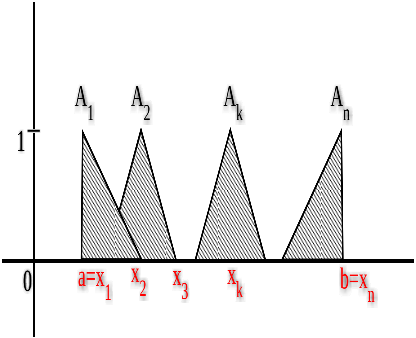

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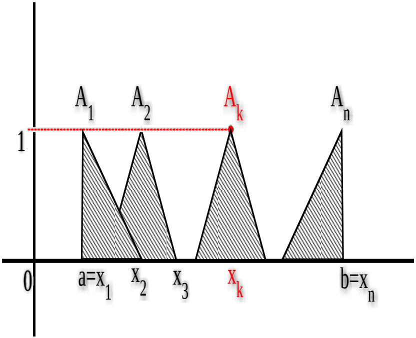

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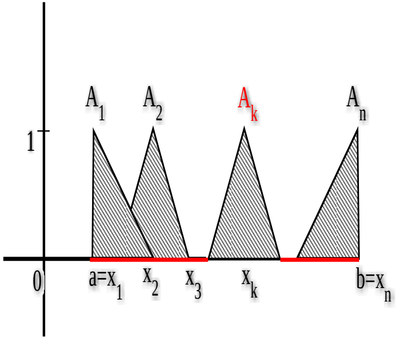

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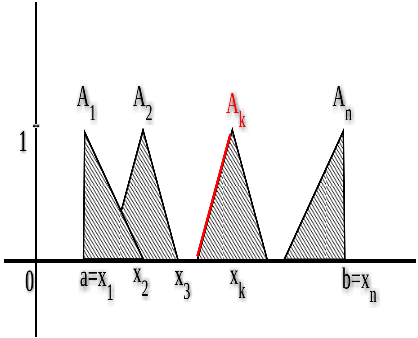

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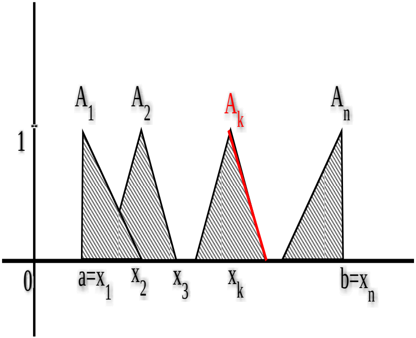

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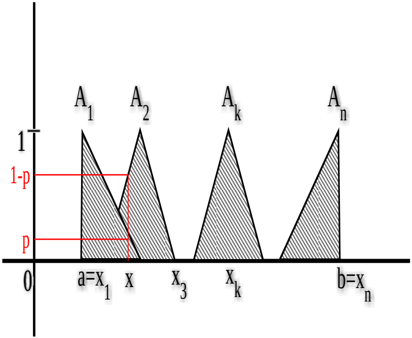

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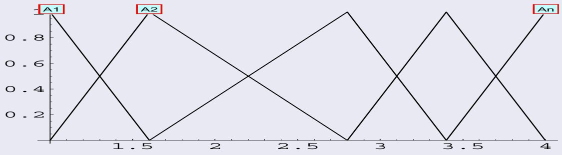

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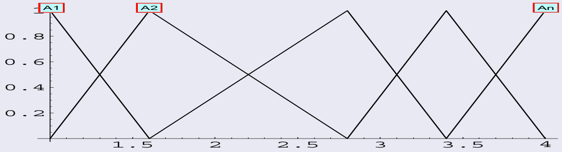

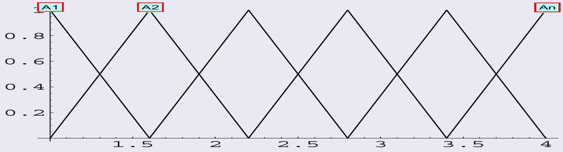

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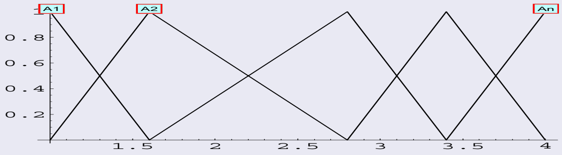
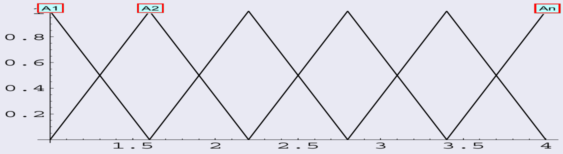
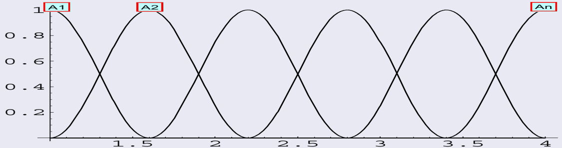
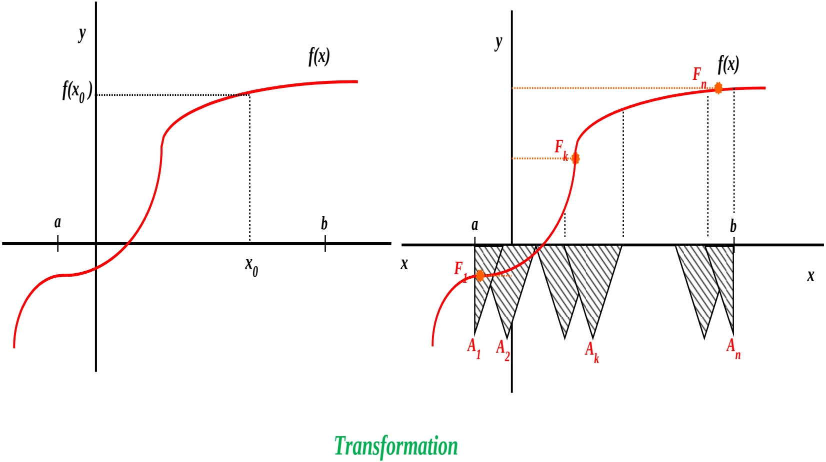






Outline

Fuzzy Transform Schematically



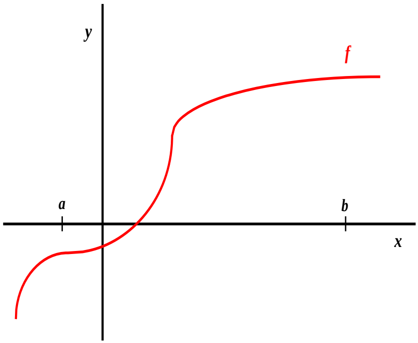
$$f: x \to f(x) \longrightarrow [F_1, F_2, \ldots, F_n]$$

Fuzzy Transform in Steps

- **Original function**
  $f : [a, b] \longrightarrow [c, d]$
- **Fuzzy partition** $A_1, \ldots, A_n$
  of $[a, b]$
- **F-Transform Components**
  $F_1, \ldots, F_n$
- **Transformation:** $f \Rightarrow$

x	A_1	A_2	$\cdots$	A_n
$\mathbf{F}_{n,f}$	F_1	F_2	$\cdots$	F_n

- **Result:** $\mathbf{F}_{n,f} = [F_1, \ldots, F_n]$



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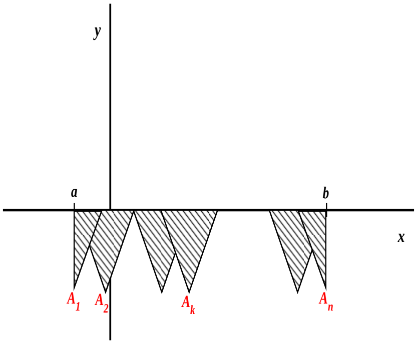

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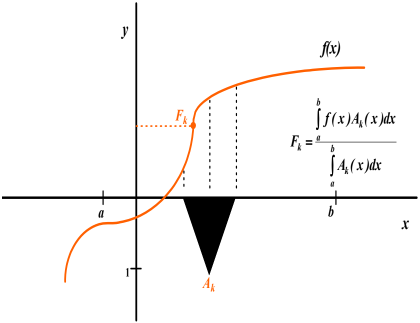

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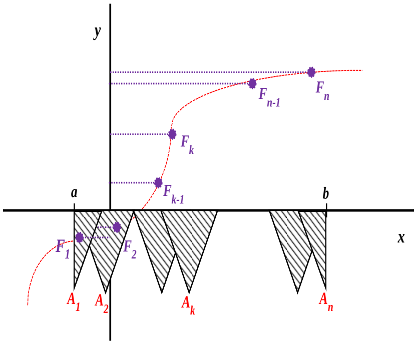

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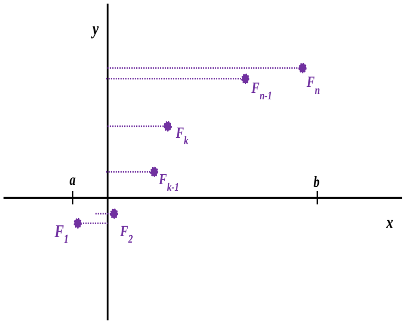


Direct Fuzzy (F)-Transform Formally

Assumptions

- $f(x) \in C[a, b]$ – continuous on $[a, b]$,
- $A_1, \ldots, A_n$ – fuzzy partition of $[a, b]$.

Definition

A vector of real numbers $[F_1, \ldots, F_n]$ is the **F-transform** of f w.r.t. $A_1, \ldots, A_n$ if

$$F_k = \frac{\int_a^b f(x)A_k(x)dx}{\int_a^b A_k(x)dx}$$

Notation: $\mathbf{F}_{n,f} = [F_1, \ldots, F_n]$

Discrete F-Transform

Definition

The vector of real numbers $[F_1, \ldots, F_n]$ is a **discrete F-transform** of f given at points $x_1, \ldots, x_l \in [a, b]$ w.r.t. $A_1(x), \ldots, A_n(x)$ if

$$F_k = \frac{\sum_{j=1}^{l} f(x_j)A_k(x_j)}{\sum_{j=1}^{l} A_k(x_j)}.$$

Inverse F-Transform

Definition

Let $[F_1, \ldots, F_n]$ be the F-transform of f w.r.t. $A_1, \ldots, A_n$

$f_{F,n}(x) = \sum_{k=1}^{n} F_k A_k(x)$ is called the **inverse F-transform**

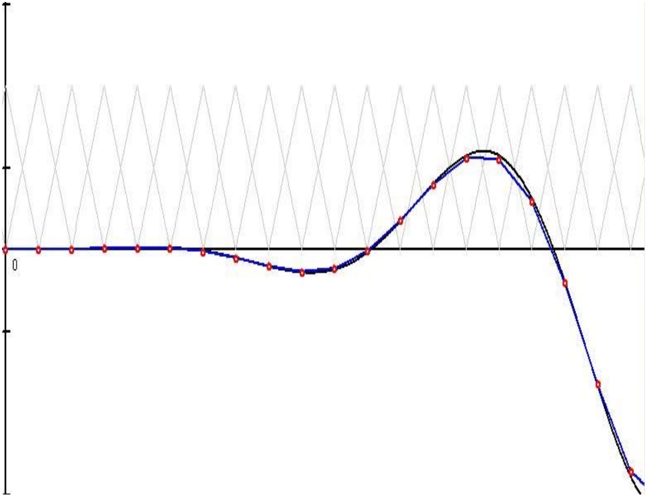
Inverse Fuzzy Transfrom. Uniform convergence

Uniform convergence

For a sequence $\{A_1^{(n)}(x), \ldots, A_n^{(n)}(x)\}$ of uniform partitions of $[a, b]$ the respective sequence $\{f_{F,n}(x)\}$ of inverse F-transforms of $f \in C[a, b]$ uniformly converges to f,

$$f_{F,n}(x) \underset{n \longrightarrow \infty}{\rightrightarrows} f(x).$$

Uniform Convergence of Inverse F-transforms



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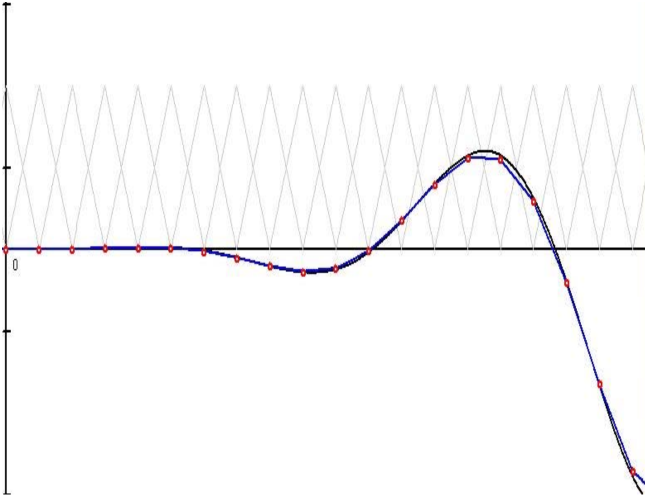

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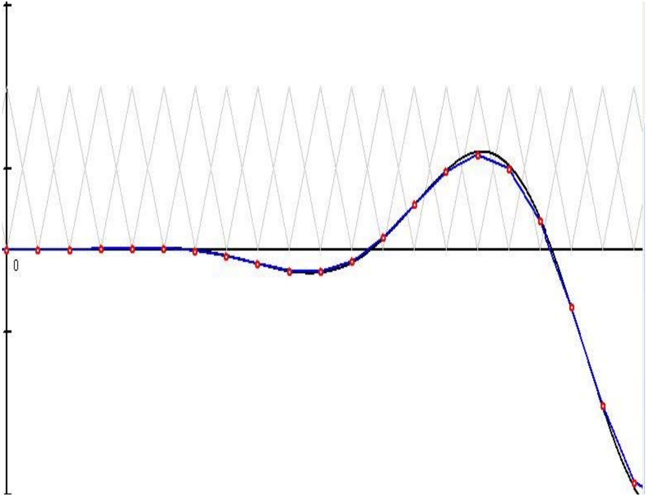

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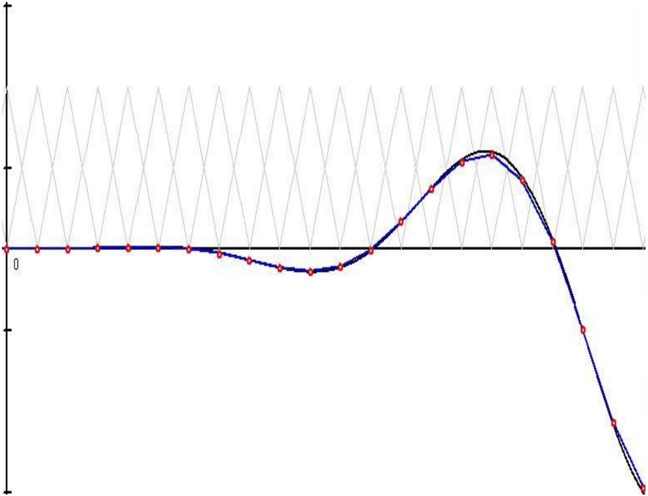

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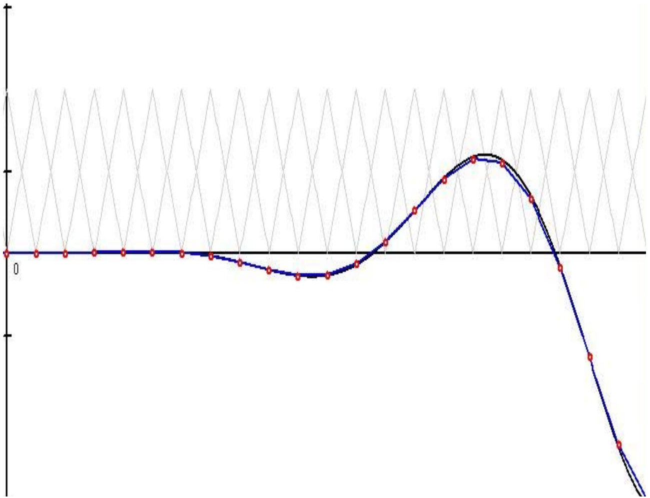

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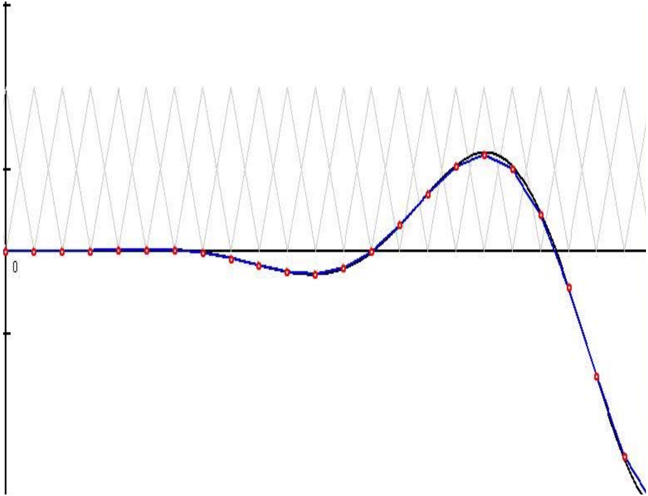

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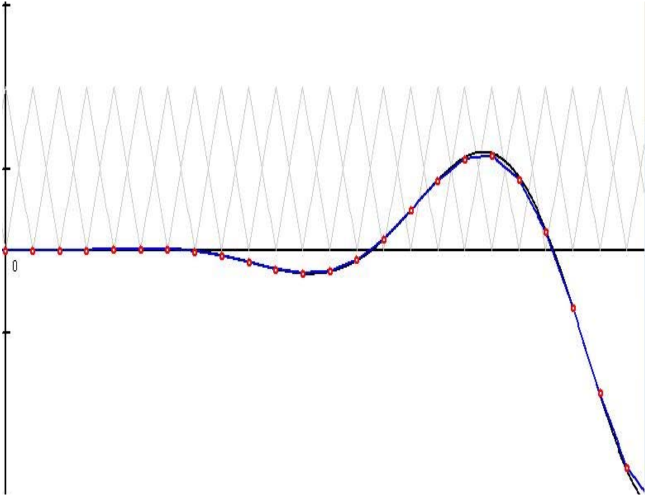

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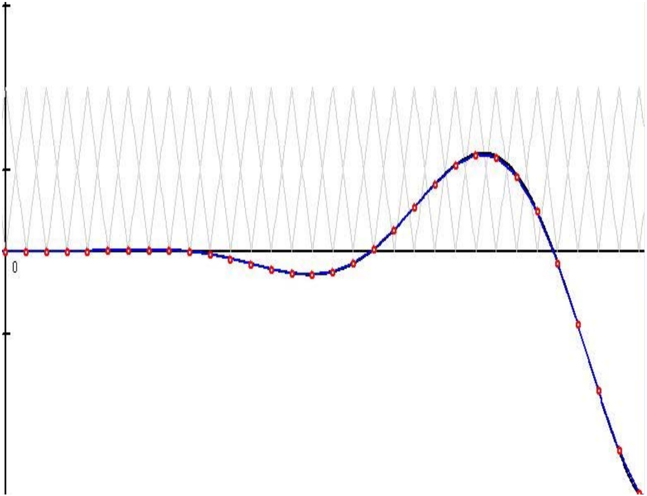
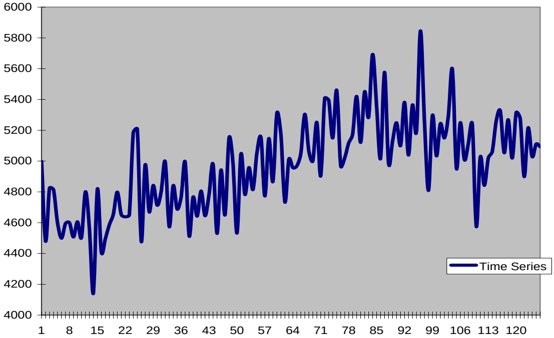


Outline

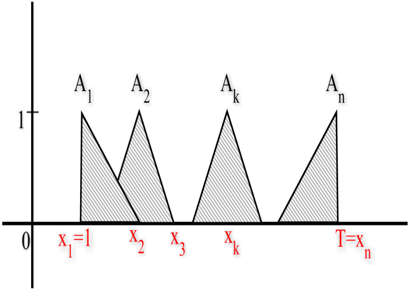
Time Series and its F-Transform

A time series x_t, $t = 1, \ldots, T$, $T \geq 3$, – **discrete function** which is defined on $S = \{1, \ldots, T\}$ so that $x : S \longrightarrow \mathbb{R}$.



Time Series and its F-Transform

Let $A_1, \ldots, A_n$, $n < T$, constitute a **fuzzy partition** of S.

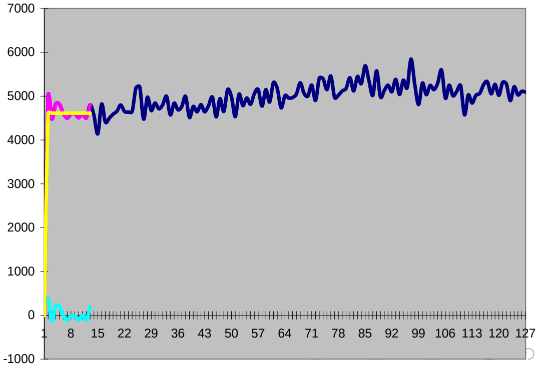


Time Series and its F-Transform

Let $(X_1, \ldots, X_n)$ – **F-transform** of x with respect to $A_1, \ldots, A_n$

Example

Component X_2
as a constant function



Decomposition of Time Series

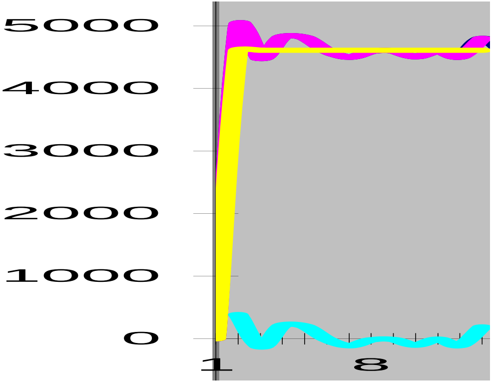
Decomposition of x_t on A_k, $k = 1 \ldots, n$

$$x_t = X_k + r_{tk}, \quad t \in A_k$$

- X_k – **k-th trend value**;
- $r_{tk} = x_t - X_k, \quad t \in A_k$ – **k-th vector of residuals**

Decomposition of x_t on A_2

- **Time series x_t on A_2**
- **FT-component** X_2
- **Vector of residuals** $r_{tk} = x_t - X_k$ **on** A_2



Synthesis of a Time Series

Synthesis

- **Reconstruction** of the time series:

$$x_t = \vee_{k=1}^{n} (X_k + \tilde{r}_{tk}).$$

Outline

Time Series Forecast

Forecast a time series –

- forecast next component X_{n+1},
- forecast next vector of residuals $r_{t,n+1}$

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FT Component Autoregressive Model

We assume that

$$X_1, X_2, \ldots, X_n$$

is an **autoregressive dynamic process** of the p-th order so that for each $k = p + 1, \ldots, n$, either we have a **linear model**

- $$X_k = \alpha_1 X_{k-1} + \cdots \alpha_p X_{k-p}$$
- or our model is a **set of fuzzy rules**

$$\text{IF } X_{k-1} \text{ is } \mathcal{A}_{11} \text{ AND } X_{k-2} \text{ is } \mathcal{B}_{11} \text{ THEN } X_k \text{ is } \mathcal{C}_1$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\text{IF } X_{k-1} \text{ is } \mathcal{A}_{l1} \text{ AND } X_{k-2} \text{ is } \mathcal{B}_{l1} \text{ THEN } X_k \text{ is } \mathcal{C}_l$$

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Vector of Residuals Forecast

- The sequence of vectors of residuals

$$(r_{t1}), \ldots, (r_{tn})$$

  is a **stationary sequence of random variables** with zero expected value.
- We assume that it is an **autoregressive dynamic process** of the q-th order so that for each $k = q+1, \ldots, n$

$$r_{tk} = \alpha_1 r_{t,k-1} + \cdots \alpha_q r_{t,k-q}$$

- Then

$$r_{t,n+1} = \alpha_1 r_{t,n} + \cdots \alpha_q r_{t,n-q+1}$$

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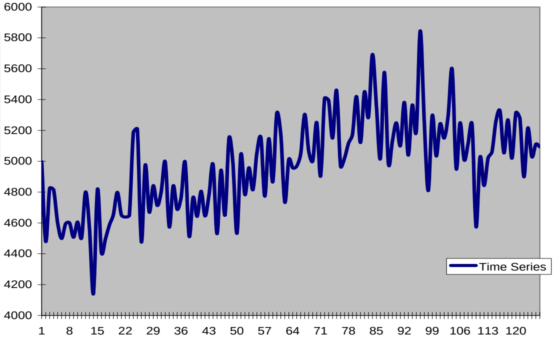
Demonstration of Time Series Analysis and Synthesis

Representation

Time series $\{x1, \ldots, x_T\}$
Function $x : [1, T] \longrightarrow \mathbb{R}$,
$T \geq 3$.

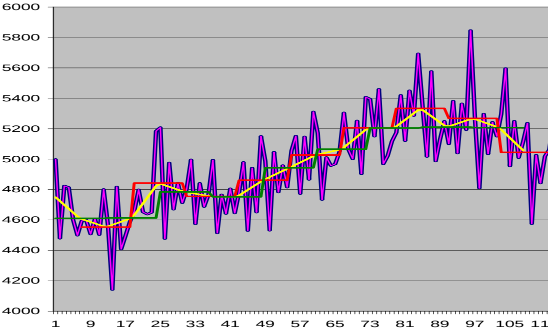


Demonstration of Time Series Analysis and Synthesis

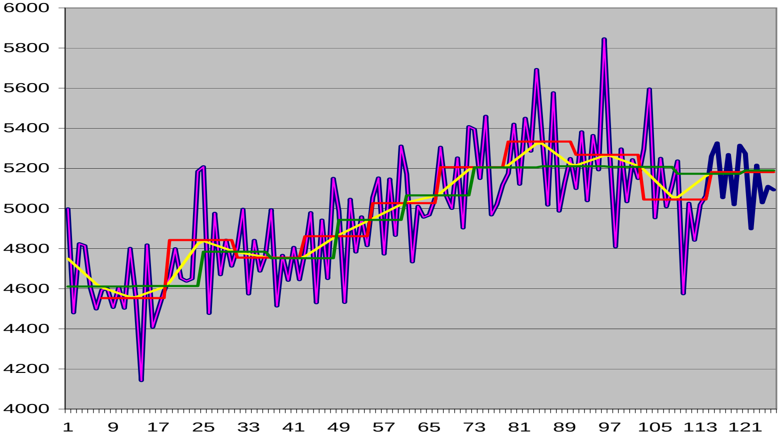
Time series trend

Trend – F-transform of x:

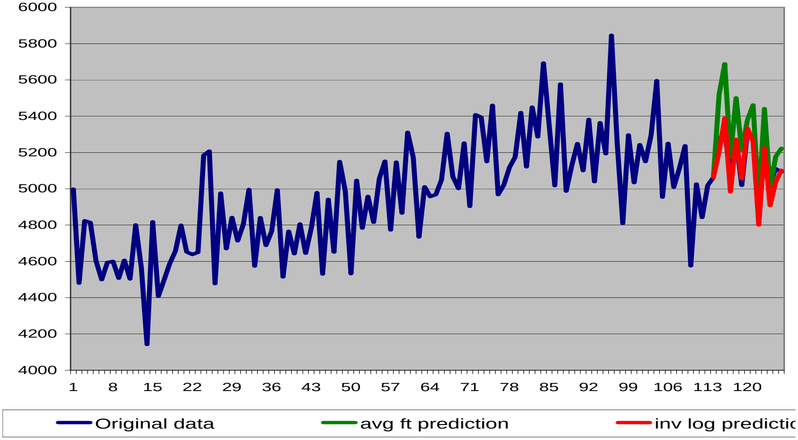
$$\mathbf{F}_n[x] = (X_1, \ldots, X_n).$$



Demonstration 1. Forecast the F-Transform Component

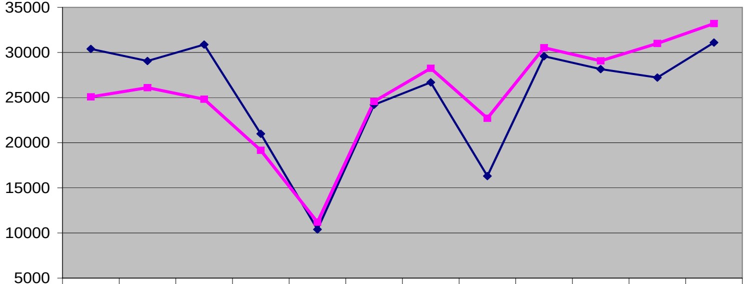


Demonstration 1. Forecast the Time Series



Demonstration 2. Comparative Testing Forecast of a Real Time Series

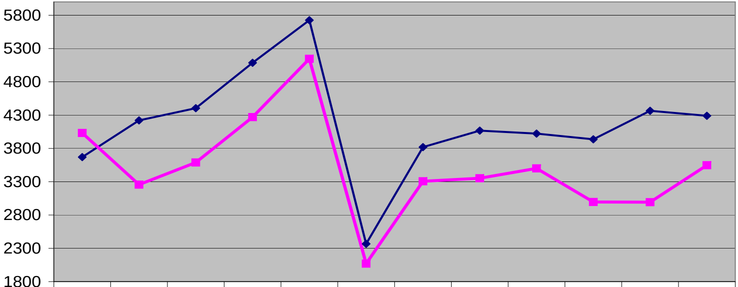
Forecast of Australia monthly production of cars and station wagons 1995



	09/94	10/94	11/94	12/94	01/95	02/95	03/95	04/95	05/95	06/95	07/95	08/95
Real data	30391	29057	30873	20991	10387	24203	26690	16308	29585	28157	27217	31101
Forecast	25080	26103	24819	19158	11192	24570	28243	22707	30523	29074	31002	33204

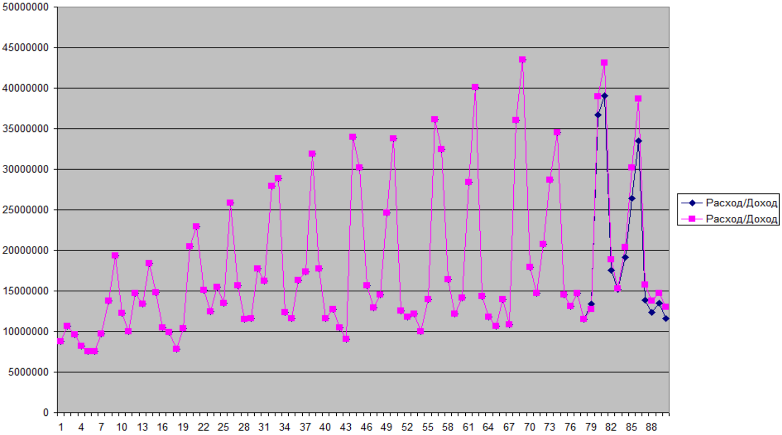
Demonstration 2. Comparative Testing Forecast of a Real Time Series

Forecast of monthly Australia sales of dry white wine (thousands of litres) 1995

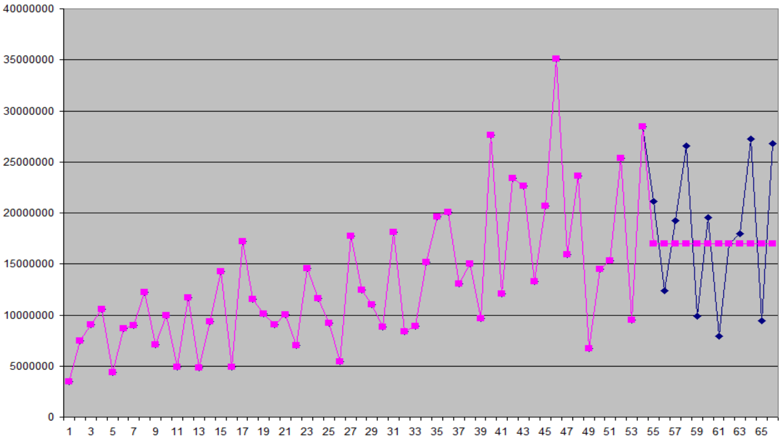


	09/94	10/94	11/94	12/94	01/95	02/95	03/95	04/95	05/95	06/95	07/95	
Real data	3670	4221	4404	5086	5725	2367	3819	4067	4022	3937	4365	4290
Forecast	4031,8	3256,2	3588,4	4269,8	5145,4	2069,8	3306,3	3352,3	3500	2995,8	2992,3	3547,9

Demonstration 3. Forecast of an Economical Time Series



Demonstration 3. Forecast of an Economical Time Series



Outline

Conclusion

- Modern Methods of Control in Economics Requires Clear Mathematical Models
- Fuzzy Modeling and Artificial Intelligence Cope with Difficulties and Vagueness of Initial Data Better than Conventional Methods
- Easiness and Effectiveness of Fuzzy Modeling Gain Advantage in Express Analysis of Economical Data

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